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# Visualization of material textures/fabrics by means of isometric Clifford Torus projections

Marc De Graef\*<sup>1</sup>

<sup>1</sup>Carnegie Mellon University – United States

## Abstract

The texture of a polycrystalline material is typically described by an orientation distribution function (ODF) that depends on the three rotational degrees of freedom. Traditionally, this function is represented graphically as a set of planar section contour plots through Euler space. More recently, 3-D visualizations using other orientation representations (Rodrigues-Frank vectors, homochoric vectors, quaternions, to name just a few) have also become available. With the recent rise in popularity of machine learning/artificial intelligence (ML/AI) approaches, the question arises of how one would go about representing a material texture so that it becomes amenable to usage in an ML/AI context. Much of the now widely available machinery of ML/AI is based on grayscale or RGB images, which are inherently 2-D in nature, so how can one convert intrinsically 3-D texture data into data that can be used as input to a neural network? In this contribution we will provide one potential answer to this question using the concept of the Clifford Torus, a mathematical object that "lives" on the quaternion unit 3-sphere. Orientation quaternions can be projected onto the torus and subsequently isometrically onto a 2-D square with periodic boundary conditions. We will show how the projection can be reformulated in terms of Rodrigues-Frank vectors which immediately leads to the insight that there are three potential projections (related to each other by a 120° rotation in Rodrigues space) that can be mapped onto the red, green, and blue channels of an RGB image. Thus, an arbitrary ODF can be represented by a 2-D RGB image, which opens the way towards the application of neural networks to texture/fabric data sets. We will show how the "square torus mapping" can be computed efficiently and how well known texture components as well as crystallographic fundamental zones can be integrated into this concept.

**Keywords:** ODF representation, texture visualization, Clifford Torus, quaternions, machine learning

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\*Speaker